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Describing the concept of *infinite* among art, literature, philosophy and science: a pedagogical-didactic overview

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Abstract

In this work an interesting overview concerning the human attempts in the description of the concept of *infinite* is presented. This peculiar concept represents a cardinal point in the history of human culture, because man, with different modalities, has always compared with it. Historically the main followed streams were two: the rational and the irrational approaches. In the first approach we find disciplines such as philosophy, mathematics and physics; the second is the domain of literature, arts and religion. Some activities for developing ideas about the intuitive concept of the infinity at the level of compulsory education will be also given.

Key words: infinite, education, pedagogy, science, philosophy, humanities, art, religion

Introduction

Introducing one of the most fascinating concepts of the human investigation, the concept of *infinite*, it is interesting to look at the meaning of the word *infinite* (Di Sia, 2013). The difficulty to deal with such concepts is verifiable also in the definitions of dictionaries; *infinite* is a non-descriptive word, often defined for negation, being composed by the prefix *in* and the substantive *finite*. It is a word which informs about itself also through opposition, a typical approach in the attempts for defining God. The so-called *negative theology* is a kind of religious and philosophical thought which aims to investigate God in a formal-logical perspective (d'Aosta, 1992); God is studied as the extreme limit beyond which logic thinking can not go, for finding the way to faith and revealed knowledge. According to the ontological arguments used by various philosophers, logical thought can not tell *what God is*, but *what God is not*. The method is known as *via negationis* (Courth, 1993) and consists of studying and defining a reality only by its opposite.

The human mind tries to approach *the absolute* thanks to the awareness of being fallible and limited; the becoming conscious of a limit is in fact a way to transcend and overcome it.

The concept of infinite is elusive for humans; already in Greek philosophy the term used for describing infinity is *apeirion*, term with the privative prefix *a* and substantive *peirar*, which means limit, border. In the millennial effort for understanding the concept, there are two basic approaches followed by humans: the rational and the irrational approaches.

Rational approaches to the concept of infinite

In the history of philosophical thought, the concept of infinite had different interpretations and developments. However, it is posible to recognise three main concepts: the negative Greek philosophy, the positive Christian thought and the concept of modern thought, especially of mathematical thinking.

The concept of infinite in Greek thought. In the Greek language, the term used for describing infinity is *apeiron*, etymologically dating back to the two terms *a* (not) and *peras* (limit). In the form *peiras* of the Ionic dialect of Miletus, it represents, according to the philosophy of Anaximander, the origin and the constituent principle of the universe (Colli, 1978; Anaximander, 1991, & Mueller, 1848). It is an infinite, unlimited, eternal, indestructible and constantly mobile matter, the first principle of all things. With Pythagoreans, a real speculation about the infinite is reached, through the association of it with the imperfection, the absence of form. This interpretation is the foundation of the concept of *horror infiniti* and is visible in the paradoxes of Zenone of Elea. The prevailing conception of infinite in the Greek world, however, is that of Aristotle, in which infinity is identified with the pure negativity of the potential matter. Epicureans, referring to the conception of Democritus, understood infinity in a positive sense, identifying it with the vacuum and as essential component of the universe in the original condition (Geymonat, 1970; Canfora, 2004).

The concept of infinite in Christian thought. Through the mediation of Neo-Platonism, Christian thought processed a positive conception of the infinity, based on the notion of God as the creator of finite realities. St. Anselm explicitly identified the divine essence with the infinite, both because this essence has no limits, and because it has an infinitely creative force. In the identity *God-Infinity* is also contained the transcendent nature of the divine, a basic concept proposed by Nicholas Cusano at the beginning of Humanism, in which there is an identification of the concept of mathematical infinite with the real infinity of God (Nagasawa, 2011; Wierenga, 1989).

The concept of infinite in Modern Thought. Starting from the ideas of Cusano, Giordano Bruno elaborated the modern version of the concept of infinite. In this version, the infinite becomes the very foundation of the universe, since the world has penetrated into every point by the creative activity of God (Di Sia, 2006; Bruno, 1998). Giordano Bruno tends to unify the actual, not only potential, way the finite with the infinite, building the starting point for the subsequent metaphysical elaborations of the *pure Ego* of Fichte, the *Absolute* of Schelling and the *Spirit* of Hegel (Weber, 2009; Falckenberg, 2013), with which the total speculative identification of a particular finite situation with the infinite is reached.

The infinite in mathematics. Near the speculative philosophical tradition, there is a concept of the infinite of a logical-mathematical kind, related to thinkers of the level of Descartes, Newton and Leibniz. It is a positive vision of infinite by considering it in a more instrumental than ontological modality; this has produced the theory of the infinitessimal calculus and the theory of limits (Di Sia, 2013; Di Sia, 2014). Subsequently, the concept of the infinite has been developed in more specialized areas, such as mathematics and formal logic, through the contributions of Gauss and Weierstrass. In modern mathematics there are different interpretations of the concept of infinite, properly addressed to a particular domain. In general, the main interpretations are related to set theory, analysis and geometry.

The infinite in set theory. Starting from the intuitive concept of a finite set, i.e. a collection in which it is possible to enumerate in finite time the elements, it is posible to define an infinite set as a *not equivalent* set to a finite set, i.e. with the impossibility to put it in bijective correspondence with the finite set (Conner, 2011). It exists also as a direct definition of an infinite set, as a set that can be put in bijective correspondence with one subset of it. (for example, the set of integers and that of even numbers). At the end of the nineteenth century, several mathematicians contributed to the extension of the concept of ordinal and cardinal numbers to infinite sets, arriving at the definition of *transfinite* numbers. The notion of a transfinite number extends the notion of number; the arithmetic operations and the order relation of natural numbers are extended to a broader class of objects with respect to the usual numbers. These entities have been introduced by Georg Cantor and serve to provide an important tool in set theory and in general in mathematics (Cantor, 2012). With these types of numbers it has been possible to categorize different types of infinite and thus to build a real arithmetic of the infinite, which turned out to be fundamental in the subsequent development of mathematics. Today the transfinite numbers are a fundamental chapter in the mathematical culture and are accepted and used, although in some cases have generated logical paradoxes of difficult solution (Dauben, 1990).

The infinite in analysis. The systematic introduction of the infinite in mathematical analysis is due to Augustin-Louis Cauchy, who defined, at the same time, also the concept of infinitessimal; it is tightly connected to the concept of *limit*. A real number l is the limit of $f(x)$, for x approaching to x_0, if the distance between $f(x)$ and l is arbitrarily small when x approaches x_0. The distance among the points is measured using the absolute value of the difference between x and x_0, i.e. $|x-x_0|$, and $|f(x)-l|$ is the distance between $f(x)$ and l. The concept of *arbitrarily small* is formally expressed through the quantifiers *for all* ($\forall$), universal quantifier, and *it exists* ($\exists$), existential quantifier.

The same procedure applies when $f(x)$ tends to positive or negative infinite, writing $\lim_{x\to x_0} f(x)=\pm\infty$. In this meaning, it does not give a definition of infinite as a number, but as a limit, i.e. the infinite is defined by means of its surroundings.

The infinite in geometry. The concept of infinite is located in the building of special geometrical situations used as a foundation. For example:

1. the *point at infinity* of a straight line is its direction, i.e. the class of parallel straight lines to the first line;
2. the *plane at infinity* of the space is the set of points and straight lines to infinite;

3. straight lines with the same point at infinity are parallel straight lines;
4. planes with the same line at infinity are parallel planes.

These concepts allow an elegant formulation of situations of parallelism between lines and planes.

Infinite and infinitessimal in physics. Contemporary physics is closely related both to the concept of infinite and of infinitessimal. By its nature, physics deals with problems on extreme dimensional scales; the physics of matter studies very small sizes, while astronomical/cosmological physics deals with huge quantities. The amazing thing is the congruence of adopted theoretical models:

1. the modern atomic model is tightly connected to the morphology of the celestial bodies;
2. the *atom-electron* structure remembers that of *planet-satellite*;
3. among the two structures there are innumerable orders of magnitude.

The exploration of the atom, initially held as indivisible, as the same name testifies, led to the discovery of smaller particles, but still divisible. Imagining repetition of the process of division so many times, the concept of physical infinitessimal can be realized; it is possible to think it as the minimum unity of matter on which the whole universe is built. Vice versa, the exploration of cosmos led to the widening of confinements of known space and to a consequent mutation in the general cosmological conception (Di Sia, 2000).

It is precisely in the different cosmological models that the concept of infinite enters with full force. The two most important studied models are:

1. the static model;
2. the inflationary model.

In the first one, the universe is understood as an entity without beginning, or end. The universe has always been as it is today; it is therefore not determined by events such as birth. In such a context there is a sort of identity between spatial infinite and temporal infinite (eternity).

In the inflationary model the universe originated from a disruptive event called the Big-Bang and this event is followed by an expansion process, still in progress.

There are also two variants of the inflationary model:

1. the expansion of the universe is seen as a never-ending process, without return, an irreversible process;
2. the expansion will continue until a critical point, after which a phase of compression of the material will begin, culminating in a new big bang.

The second kind of expansion is called also *the pulsating universe*; it shares with the static model the key concept that the life of the universe is eternal and the universe extends itself in space to infinity. In the static model this happens for *datum of fact*, while in the pulsating model it happens through cycles of compression--expansion, eternally repeating.

Therefore the infinite, under various forms, is inside the reality to all levels and to all scales; by the infinitely great (real infinity) to the infinitely small (infinitessimal) (Di Sia, 2001; Albeverio & Blanchard, 2013).

Not rigorously rational approaches to the concept of infinite

In the products of art, the concept of infinity finds great expression; a lot of artists have confronted the reality of it. In this paper we examine in particular the areas of poetry, literature and graphic arts.

The infinite in poetry. The comparison with the infinite in the poetry finds a milestone in one of the greatest Italian poets, Giacomo Leopardi, who wrote a poem just titled *The Infinite*. Leopardi attempts a description of the infinite through a connection, putting the infinite in relation to what is known, finite; the description of the infinite results from opposition. The infinite is described as that, which is beyond the hedge, going beyond our physicality:

> "Sempre caro mi fu quest'ermo colle, e questa siepe, che da tanta parte dell'ultimo orizzonte il guardo esclude" (Leopardi, 2013)
> "Always dear it was to me this lonely hill, and this hedge, that from so many parts of the last horizon excludes the view".

The last horizon represents the true limit of the human nature with respect to the universe. It is the last, in the sense of not surmountable, not crossable, therefore not entirely understandable. With this comparison the poet gains a partial attainment of the infinite and reaches a new dimension:

> "e mi sovvien l'eterno, e le morte stagioni, e la presente e viva, e il suon di lei" (Leopardi, 2013)
> "and the eternity helps me, and the death seasons, and the present and live one, and its sound".

Opposite however is the interpretation given by the poetry of Giovanni Pascoli in the poem *The vertigo*. In his vision, the perspective is no longer *geocentric*, there is no comparison with the finite, but there is the suggestion proposed by the infinite spaces, by the astronomical dimensions:

> "Qual freddo orrore pendere su quelle lontane, fredde, bianche azzurre e rosse, su quell'immenso baratro di stelle, sopra quei gruppi, sopra quelli ammassi, quel seminìo, quel polverìo di stelle!" (Pascoli, 2012).
> "What cold horror to hang on those distant, cold, whites, blues and reds, on those immense abysses of stars, above those groups, above those heaps, seeding, powder of stars!".

The entire second part of the poem is based on a cosmological vision of the infinite and the poet is completely dominated and lost; not surprisingly, he describes the infinite with the expression *cold horror*. The two poems, however, have a common feature: in both poets there is the perception of a pleasure given by collapsing into the *unknown*. Leopardi says:

> "Così tra questa immensità s'annega il pensier mio: e il naufragar m'è dolce in questo mare" (Leopardi, 2013)
> "So in this immensity my thought is drowned: and the shipwreck is sweet in this sea".

Similarly Pascoli, on the end of the work, writes:

"precipitare languido, sgomento, nullo, senza più peso e senza senso. Sprofondar d'un millennio ogni momento!" (Pascoli, 2012).
"to precipitate languid, dismay, nothingness, without weight and meaningless. To collapse of a millennium at any instant!".

For the first poet there is a sweet shipwreck in the sea of the unknown, for the second a languid fall in the cosmic space. We have therefore two very different visions: the man, though frightened by the unknown, seeks to compare himself with it, enjoys thinking of the infinite.

The infinite in literature. Even in literary works in prose the infinite was considered. In various forms, several writers compared with this concept and produced different and imaginative interpretations. The English writer Laurence Sterne, in his masterpiece *The Life and Opinions of Tristram Shandy, Gentleman*, or briefly *Tristram Shandy*, (Sterne, 1992) describes the infinite in a very similar way to Zenone's paradox of Achille and the turtle. He takes one year to describe a single day in his own life, leaving so irremediably late. Therefore the work is impossible to finish, it is a kind of infinite regression that will never allow completion.

Different is the reading given by the Argentine writer Jorge Luis Borges in the short story *The Library of Babel* (Borges, 1944); he talks about the existence of a book in a library, in which all books are listed, a kind of large index of the library, itself written in a book. The paradox is obvious: for completing the indexing of volumes, it would be necessary to launch a new book, listing the first index and then again *ad infinitum*.

One of the most special literary interpretations is present in the work of the Irish writer James Joyce. His whole work, represented emblematically by *Ulysses* (Joyce, 2013), is the description of the infinite tortuous movements of the human mind. Ulysses is an ordinary person and his wandering is not geographical, as in the Homeric version, but mental. The book describes the infinite stream of thoughts, the *stream of consciousness* of the protagonist, creating a very intricate labyrinth in which the reader is thrown.

Graphical representations of the infinite. The infinite has been described in various ways in the graphic arts. One of the most interesting visions is that proposed by the Dutch engraver Mauritius Cornelius Escher. He looked at the infinite in two different ways:

1. the infinite divisions of the plane;
2. the infinite movements of the space.

In the first way we find works as in figure 1 (Taylor, 2009).

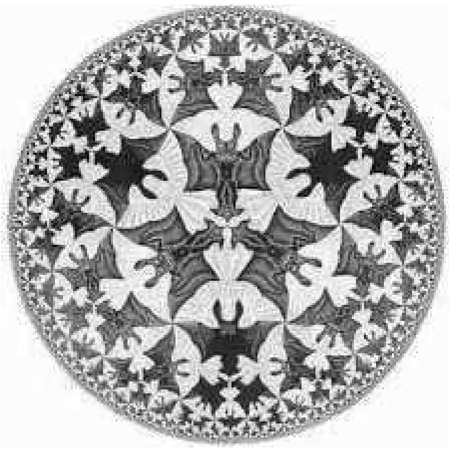

Figure 1. Circle limit IV (M. C. Escher)
Source: http://www.mcescher.com/.

Escher creates a special geometry in which the visual plane is divided into smaller and smaller sections, always equal to themselves. In figure 1 the fragmentation occurs to the outer edge, but in works as *Smaller and Smaller* (figure 2) this happens in reverse, towards the centre.

To the second set belong works such as "*Ascending and Descending*" (figure 3).

In this famous lithography, Escher intends to represent a physical paradox in which, through an optical effect, the men represented in the upper part go up (or down) indefinitely. Still infinite motion is represented, although in another sense, in works as *Moebius Strip II* (figure 4). There is no solution of continuity between the internal and external dimensions of the figure; it is not posible to distinguish between an *inside* and an *outside*.

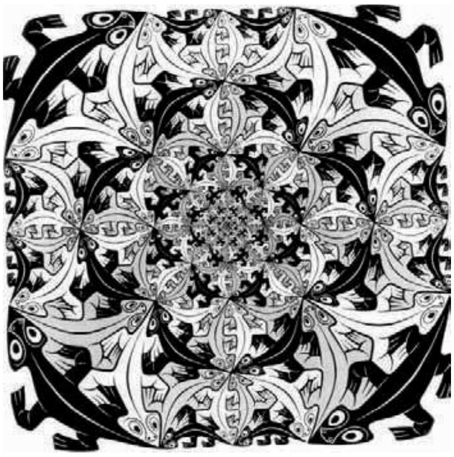

Figure 2. Smaller and Smaller (M. C. Escher)
Source: http://www.mcescher.com/.

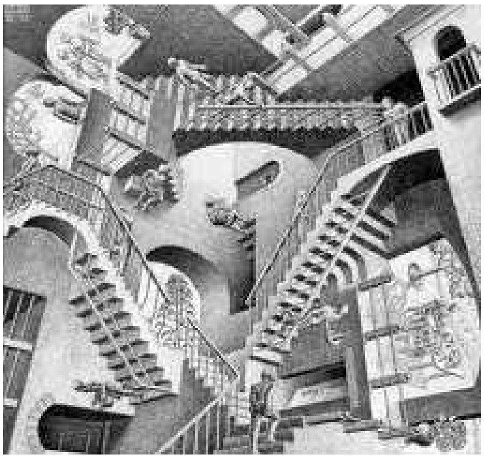

Figure 3. Ascending and Descending (M. C. Escher)
Source: http://www.utwente.nl/ewi/trese/.

Figure 4. Moebius Strip II (M. C. Escher)
Source: http://www.mcescher.com/.

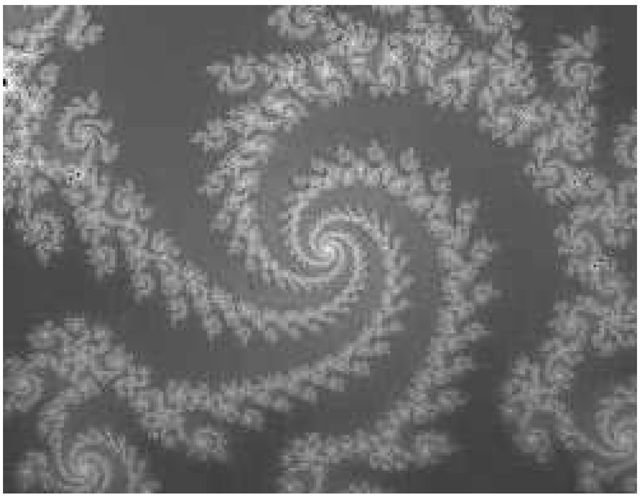

Figure 5. A fractal
Source: http://www.math.harvard.edu/.

Also particular mathematical constructions can generate interesting figures, in which the concept of infinite is implicit. In fractals, particular figures introduced by the mathematician Benoît Mandelbrot, we find a division of the plan, which is very similar to that imagined by Escher (figure 5).

A fractal is a geometric object, which is endowed of internal homothety, i.e. repeats itself in its form on different scales. The term *fractal* was coined by Mandelbrot in 1975 (Di Sia, 2013; Di Sia, 2014; Mandelbrot, 1982; Mandelbrot, 2004, & Pickover, 2001), for describing some mathematical behaviour that seemed to be *chaotic*. It derives from the Latin *fractus*, broken, as well as the mathematical term *fraction*. Nature produces many examples of forms, which are very similar to fractals; for example, in trees, especially in firs, each branch is approximately similar to the whole tree, each small part of branch is similar to its own branch, and so on. Fractals are also present in the geomorphology of mountains, in clouds, in ice crystals, in some leaves and flowers. The *fractal art* is created by calculating fractal mathematical functions and transforming the results of calculations into pictures, animations, music and other forms of artistic expression.

The religious tension toward the infinite. Sometimes the advanced theoretical models of science reflect ancient beliefs of man and religion; often the relationships between science and religion have been not conflictual, but complementary. For example, in the cosmological vision of the inflationary model, the universe is not eternal. The universe is born through a catastrophic event, develops and, with all probability, dies. Such conception is near to the cosmological model proposed by the Christianity. The beginning of the holy Bible, *Genesis*, states: "At the beginning God created the sky and the earth" (Gn 11).

The end of the Bible, *Apocalypse*, states that the world will end with the "Universal Judgment", at the end of times (Ap 1616) (The Holy Bible, 1993). So, for the Christian religion, space and time are finite, as for the inflationary model.

Other religions intend instead the time as infinite and therefore are near to the static model. In Hinduism, the three Gods Brahma, Vishnu and Shiva create, hold in life and destroy cyclically the universe, a very similar conception to the pulsating model (Flood, 1996; Zimmer, 1972).

In conclusion, the monotheist religions (Judaism, Christianity, Islam) intend the universe as finite and as product of a creative event of God; in the polytheist religions, instead, the universe is understood as eternal and it is not directly connected to the generative intervention of a divinity.

The infinite at the level of compulsory education

The intuitive knowledge of students, acquired in extra-scholastic contexts or in previous studies, can facilitate or obstruct in a decisive way the learning of a concept. Fischbein refers to such intuitive ideas as to *primary intuitions* and underlines the opportunity to strengthen them; so they can evolve as *secondary intuitions*, constituting a propitious ground for the acquisition of the concepts (Fischbein, 1973; Fischbein, 1987). These results are particularly important in the case of the most complex and delicate concepts of mathematics, as the concept of infinite and those tightly connected of infinitessimal, limit and continuity (Di Sia, 2013; Di Sia, 2014). About the difficulties connected with such concepts, the presence of real *epistemological obstacles* has been underlined (Di Sia, 2013; Di Sia, 2014; Brousseau, 1983; Sierpinska, 1985, & Sierpinska, 1987), suggesting the opportunity of particular didactic choices (Grugnetti-Rizza et al., 1998; Hauchart & Schneider, 1996). Difficulties of linguistic nature are also present, due to the verifiable ambiguities of the daily use of terms as *limit* and *infinite* (Grugnetti-Rizza et al., 1998; Alberti-Andriani et al., 2001).

With the purpose of exploring the intuitive knowledge of students of primary and secondary school, related to the concepts of infinite and infinitessimal, research showed the presence of primary intuitions at all considered age levels. On the other hand, if not adequately considered, these insights generally tend to regress in the course of the school years. The teaching practice seems to have mainly the effect of moving the attention more on calculation than on reasoning (Di Sia, 2013; Di Sia, 2014, & Andriani et al., 1998).

The *basis* mathematics is very rich in possibilities for gradually familiarizing with the concept of infinite; the mandatory school can have a fundamental influence in the development of favorable mental images of infinite. As example, we remember the question of the division by zero; it is often quickly resolved as not motivated prohibition, which risks to produce distorsions and errors. It could become on the contrary an opportunity for speaking of *numbers growing over every fixed limit* and for arousing questions and curiosity about the infinite (figure 6).

We calculate $\frac{7}{0}$. The result is……?

Try to calculate $\frac{7}{0.1}, \frac{7}{0.01}, \frac{7}{0.001}, \frac{7}{0.0001}$ …… and think about.

Figure 6. Thinking about the concept of infinite and infinitessimal.
Source: personal elaboration.

In textbooks, a confrontational attitude is often present; some authors carefully avoid terms such as *infinite*, *limit*, *continuous*, others use expressions such as *to be satisfied of such approximation for…*, *to go close to…*. In this way, the deep sense of the problem is lost, so as the possibility of a right comprehension of the approximation.

A matter constituting one of the first occasions for approaching the concept of infinite and the idea of limit, are the geometric progressions; students meet them already in the mandatory school. The convergence and divergence, in relation to the value of the *reason* of the progression, represents an interesting window on the problem of infinite. Particular interest have sums of numbers in geometric progression, as for example:

$$S_1 = 1 + 1/2 + 1/4 + 1/8 + \ldots\ldots \text{ (convergent sum)},$$

or:

$$S_2 = 1 + 2 + 4 + 8 + \ldots\ldots \text{ (divergent sum)}.$$

It is possible, starting in the mandatory school, to realize cards in which the request is to imagine the final behaviour of a particular sequence of geometric figures. The purpose of cards is also the understanding of the idea of *iteration*, so as the difference between convergent and divergent processes (figure 7).

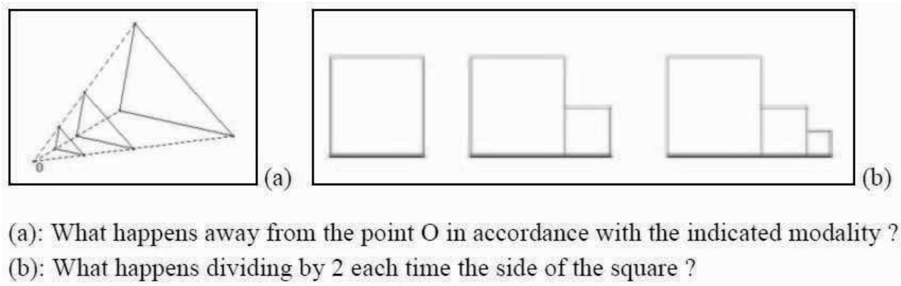

Figure 7. Convergent and divergent processes
Source: personal elaboration.

Conclusions

The concept of infinite is intimately connected to the development of humanity, in all intellectual, artistic, scientific expressions. Such a concept is therefore a limit, a border of the human generation, intrinsically opposite to that of infinite. Through this comparison, humanity better understands its own state, delineates its own nature. With investigation on the infinite, humanity has understood during the course of history a lot of dynamics and resolved many problems.

There exists a metaphor, in which knowledge is compared to an island and ignorance to the border of the island with the sea, i.e. with the *unknown*. Increasing knowledge, i.e. the island, ignorance also increases; but for geometric reasons, ignorance grows in linear way, while knowledge grows in a quadratic way, being an area. The knowledge of infinite has certainly created problems, but at the same time has increased and currently increases the knowledge of humanity, making possible such new knowledge for physics, mathematics and in general for the pure thought.

Complex concepts, as that of infinite, can be deeply understood only as an immense task. Beginning from the most spontaneous and immediate intuitions, it can create in every child, which will become an adult, a coherent and solid picture of mental images.

The exploration of the infinite leads also students to reason in terms of approximation, accuracy and control of mistakes (Di Sia, 2013; Di Sia, 2014).

References

Alberti, N., Andriani, M. F., Bedulli, M., Dallanoce, S., Falcade, R., Foglia, S., Gregori, S., Grugnetti, L., Marchini, C., Molinari, F., Pezzi, F., & Rizza, A. (2001). *On the difficulties of learning the concept of limit*. Parma: Riv. Mat. Univ. Parma, 6(3), 1-21.

Albeverio, S., & Blanchard, P. (Eds). (2013). *Direction of Time*. New York: Springer.

Anassimandro of Mileto (1991). *Fragments and testimonies*. Paris: Presses universitaires de France.

Andriani, M. F., Dallanoce, S., Grugnetti, L., Molinari, F., & Rizza, A. (1999). Autour du concept de limite. In: F. Jaquet (Ed.), *Proceedings of CIEAEM 50,* Neuchâtel 2-7 August 1998, 329-335.

Borges, J. L. (1944). *Ficciones (1935-1944)*. Buenos Aires: SUR.

Brousseau, G. (1983). *Epistemological obstacles and problems in mathematics*. Research in Mathematics Education, 4 (2), 165-198.
Bruno, G. (1998). Cause, Principle and Unity: And Essays on Magic. *Series in Cambridge Texts in the History of Philosophy*. Cambridge: University Press.
Canfora, L. (2004). *We and the ancients. Why to study Greeks and Romans is useful for the intelligence of modern*. Rizzoli, Milano: BUR Biblioteca University.
Cantor, G. (1955). *Contributions to the Founding of the Theory of Transfinite Numbers*. New York: Dover.
Colli, G. (1978). *Greek wisdom II - Epimenide, Ferecide, Talete, Nico, Dario, Ennio, Anassimandro, Anassimene, Onomacrito*. Milano: Adelphi.
Conner, J. (2011). *On the Logical Form of Correlatively Infinite Sets*, Bloomington: Xlibris Corp.
Courth, F. (1993). *The mystery of God Trinity*. Milano: Editoriale Jaca Book.
D'Aosta, A. (1992). *Proslogion*. Milano: BUR Biblioteca University.
Dallanoce, S., Grugnetti, L., Molinari, F., Rizza, A., Andriani, M. F., Foglia, S., Gregori, S., Marchini, & C., Pezzi, F. (2000). *A cognitive co-operation across different sectors of education*. In: Ahmed, A., Kraemer, J. M., and Williams, H. (Eds.). *Proceedings of conference CIEAEM 51*. Chichester: Horwood Publishing, 297-303.
Dauben, J. W. (1990). *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton: Princeton University Press.
Di Sia, P. (2000). The role of space and time in the evolution of unified theories (part 1). *MatematicaMente,* 25, 1.
Di Sia, P. (2000). The role of space and time in the evolution of unified theories (part 2). *MatematicaMente*, 27, 1-2.
Di Sia, P. (2000). The role of space and time in the evolution of unified theories (part 3). *MatematicaMente,* 28, 2.
Di Sia, P. (2001). The first second of the Universe. *MatematicaMente,* 40, 1-2.
Di Sia, P. (2006). Time, man, God: Reflections (part 1). *MatematicaMente*, 99, 1-2.
Di Sia, P. (2006). Time, man, God: Reflections (part 2). *MatematicaMente,* 100, 2.
Di Sia, P. (2013). *Elements of Mathematics Education I - Laboratory*. Roma: Aracne Editrice.
Di Sia, P. (2013). *Foundations of Mathematics and Education I*. Roma: Aracne Editrice.
Di Sia, P. (2014). *Foundations of Mathematics and Education II*. Roma: Aracne Editrice.
Falckenberg, R. (2013). *History of Modern Philosophy: From Nicolas of Cusa to the Present Time*. Seattle: CreateSpace Independent Publishing Platform.
Fischbein, E. (1973). Intuition, structure and heuristic methods in teaching Mathematics. In: A. G. Howson (Ed.). *Developments in Mathematical Education*. Cambridge: Cambridge University Press, 222-232.
Fischbein, E. (1987). *Intuition in Science and Mathematics*. Dordrecht: D. Reidel Publishing Company.
Flood, G. (1996). *An Introduction to Hinduism*. Cambridge: Cambridge University Press.
Geymonat, L. (1970). *History of philosophical and scientific thought*. Milano: Garzanti.
Grugnetti, L., Rizza, A. & al. (1999). The limit concept: what is the relationship with the natural language? In: F. Jaquet (Ed.). *Proceedings of conference CIEAEM 50,* Neuchâtel 2-7 August 1998, 313-318.
Hauchart, C., Schneider, M. (1996). A heuristic approach to Analysis. *Repères quarterly journal*, Aix-Marseille: IREM University, 25, 35-60.
Joyce, J. (2013). Ulisse. In: G. Celati (Ed.), *Letture*. Torino: Einaudi.
Leopardi, G. (2013). *Opere*. Torino: UTET.
Mandelbrot, B. B. (1982). *The Fractal Geometry of Nature*. Cranbury: W. H. Freeman and Company.
Mandelbrot, B. B. (2004). *Fractals and Chaos: The Mandelbrot Set and Beyond*. New York: Springer.
Mueller, C. (1848). *Fragmenta Historicorum Graecorum*. Paris: Parisiis.
Nagasawa, Y. (2011). *The Existence of God*. London: Taylor & Francis.
Pascoli, G. (2012). *Various poems.* London: Ulan Press.
Pickover, C. A. (2001). *Computers, Pattern, Chaos and Beauty*. New York: Dover Publications.
Sierpinska, A. (1985). Epistemological obstacles related to the notion of limit. *Recherches en Didactique des Mathématiques,* 6 (1), 5-67.
Sierpinska, A. (1987). Humanities Students and Epistemological Obstacles Related to Limits. *Educational studies in Mathematics*, 18 (4), 371-397.
Sterne, L. (1992). *Life and Opinions of Tristram Shandy, Gentleman*. Milano: Grandi Classici Mondadori.
Taylor, R. (2009). Reflecting the impossible. *Nature*, 460, 462.
The Holy Bible: New International Version (1993). Red Lion: HarperTorch.
Weber, A. (2009). *History of Philosophy*. Charleston: BiblioBazaar.
Wierenga, E. R. (1989). *The Nature of God: An Inquiry Into Divine Attributes*. Ithaca: Cornell University Press.
Zimmer, H. (1972). *Myths and Symbols in Indian Art and Civilization*. Princeton: Princeton University Press.